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Modeling Infiltration during a Steady Rain

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Few of the infiltration models in current use are suitable for the situation in which the rainfall intensity is initially less than the infiltration capacity of the soil. In this paper a simple two-stage model is developed for infiltration under a constant intensity rainfall into a homogeneous soil with uniform initial moisture content. The first stage predicts the volume of infiltration to the moment at which surface ponding begins. The second stage, which is the Green-Ampt model modified for the infiltration prior to surface saturation, describes the subsequent infiltration behavior. A method for estimating the mean suction of the wetting front is given. Comparison of the model predictions with experimental data and numerical solutions of the Richards equation for several soil types shows excellent agreement.

There has been great interest during the past decade in mathematical modeling of the watershed rainfall-runoff process, and this interest is expected to continue. Many of the watershed models have been formulated by combining models that represent the actual components of the hydrologic cycle, such as infiltration, overland flow, and evapotranspiration [e.g., *Crawford and Linsley*, 1962; *Huggins and Monke*, 1966]. Of the many components, infiltration has the largest influence on the volume of watershed runoff. For the continental United States, >70% of the annual precipitation infiltrates into the soil [*Chow*, 1964], although this percentage varies widely for individual storms. Yet despite the importance of infiltration most models of the infiltration process have serious deficiencies in their representation of infiltration from rainfall.

For many rainfall events there is an initial period during which all the rainfall infiltrates into the soil. During this time, as water infiltrates, the capacity of the soil to absorb water decreases until it becomes less than the rainfall intensity. At this point, water begins to accumulate on the soil surface, and runoff can begin. To properly represent a runoff event, the hydrologist must be able to predict this time and also the subsequent decline in infiltration capacity.

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The process of infiltration is very complex, even when it is assumed that the soil is a homogeneous medium with a uniform initial moisture content. For the work presented here, additional assumptions are that the rainfall is of constant intensity until runoff begins and that the infiltration process at a given location is one dimensional. Raindrop compaction is not considered.

INFILTRATION BEHAVIOR

One can consider three distinct cases or stages of infiltration when a rainfall of intensity I is applied to a soil having a saturated conductivity K_s and an infiltration capacity f_p.

Case A: $I < K_s$. For this condition, runoff will not occur, since all the rainfall infiltrates. In a watershed model, however, the rainfall must still be accounted for because the soil moisture level is being altered. Line A in Figure 1 shows this situation.

Case B: $K_s < I \leq f_p$. During this stage, all the rainfall infiltrates into the soil, and the soil moisture level near the soil surface increases. Line B of curve BC in Figure 1 illustrates this case.

Case C: $K_s < f_p \leq I$. The infiltration rate is at capacity and decreasing. Runoff is being generated. This is the stage shown by curve C and also curve D in Figure 1.

Most field experiments and prediction equations have been concerned with case C only, since they generally assume an excess supply of water at the soil surface from time 0 and de-

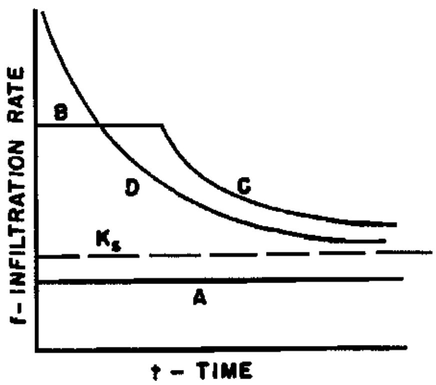

Fig. 1. Different cases of infiltration behavior under rainfall.

scribe the behavior in the form of a curve such as curve D in Figure 1. The study reported here is concerned with events involving both cases B and C, since, in the opinion of the authors, case B usually precedes and influences the infiltration behavior in case C.

EXISTING INFILTRATION MODELS

There are several models commonly used to simulate infiltration. The best known are the empirical equations of Kostiakov and Horton [*Childs*, 1969], which have been popular because of their simplicity and capability of being fit to most infiltration capacity data. However, both equations contain parameters that are difficult to predict because they have no physical significance.

A more recent empirical equation or model is that given by *Holtan* [1961], which expresses the infiltration capacity as a function not of time but of the unoccupied pore space in the soil. A model of this type is convenient for a watershed model, but determining the control depth is uncertain [*Huggins and Monke*, 1966].

For a homogeneous soil with an excess supply at the surface, *Philip* [1957] derived an infiltration equation with predictable parameters. Unfortunately, computing these parameters is difficult [*Whisler and Bouwer*, 1970], and their values are more commonly obtained by fitting. A further difficulty is the assumption of an excess water supply at the surface; thus an event beginning as case B cannot be represented.

A simple equation proposed by *Green and Ampt* [1911] has been the focus of renewed interest. It can be written as

$$f_p = K_s[1 + (M_d \cdot S/F)] \qquad (1)$$

where

f_p, infiltration capacity, in./hr or cm/sec;
K_s, saturated conductivity, in./hr or cm/sec;
M_d, initial moisture deficit for the range of moisture content $\theta_s - \theta_i$, volume/volume;
S, capillary suction at the wetting front, in. or cm of water;
F, cumulative infiltration from the beginning of the event, in. or cm.

Equation 1 was derived by applying Darcy's law to the situation of infiltration from an excess surface water supply from time 0. The variables are all predictable, since they have physical significance, although determining S has caused some difficulties. *Bouwer* [1966] assumed it to be the water-entry value, which was reported to be approximately half the air entry value. *Philip* [1957] took it to be the height of capillary rise in the soil.

The Green and Ampt equation has given good results when it has been applied to nonuniform profiles that become denser with depth [*Childs and Bybordi*, 1969] and to the case of a partially sealed surface [*Hillel and Gardner*, 1970]. Computation for nonuniform moisture content is possible also [*Bouwer*, 1969].

A combination of Darcy's law as applied to unsaturated flow and the equation of continuity results in the second-order nonlinear partial differential equation for one-dimensional vertical moisture movement, sometimes referred to as the Richards equation [e.g., *Childs*, 1969]. Although this equation is not suitable for general application, it is considered the best method available for computing vertical flow of soil moisture and was therefore used as a working basis for this study.

$$\frac{\partial \theta}{\partial t} = -\frac{\partial}{\partial z}\left(K(\theta)\,\frac{\partial S(\theta)}{\partial z}\right) - \frac{\partial K(\theta)}{\partial z} \qquad (2)$$

where

θ, volumetric moisture content;
t, time;
z, distance below the surface;
$S(\theta)$, capillary suction;
$K(\theta)$, unsaturated conductivity.

For the general case, (2) must be solved numerically, and the relationships of conductivity and suction versus moisture content must be known. The numerical solution is rather complex, and the required soil data are difficult to obtain. It is, however, theoretically sound, and many tests against field trails have shown good agreement [e.g., *Nielsen et al.*, 1961; *Whisler and Bouwer*, 1970; *Wang and Lakshminarayana*, 1968].

DEVELOPMENT OF MODEL

Case B: infiltration prior to runoff. As was described earlier, the moisture content at the surface increases during rainfall until surface saturation is reached. Although Darcy's law is applicable at any time during the process, it is at the moment of saturation that a useful relationship can be developed, for at that moment the surface moisture content and conductivity are known.

The moisture content profile at the moment of surface saturation is approximately as shown in Figure 2*a*. The area above the new moisture profile is the amount of infiltration up to surface saturation F_s. The shaded area is drawn equal to this area, a saturated zone of depth L_s thus in effect being substituted for the actual moisture profile. The two areas are by definition equal and given by

$$F_s = M_d \cdot L_s \qquad (3)$$

In finite difference form, Darcy's law can be written as

$$q = -K(\theta)(\Phi_2 - \Phi_1)/(z_2 - z_1) \qquad (4)$$

where q is the flow rate, $K(\theta)$ is the capillary conductivity, Φ is the total potential, and z is the distance below the surface. The subscripts 1 and 2 refer to the surface and the wetting front, respectively. Applying (4) to Figure 2*a*, we see that $z_2 - z_1 = L_s$. If we take the potential at the surface Φ_1 as 0, $\Phi_2 = -(L_s + S_{av})$, where S_{av} is the average capillary suction at the wetting front (discussed later). At this moment of surface saturation the infiltration rate is still equal to the rainfall intensity, so that $q = I$. The capillary conductivity can be assumed to be equal to the saturated conductivity K_s. Making these substitutions in (4), we obtain

$$I = K_s(S_{av} + L_s)/L_s \qquad (5)$$

If we combine (3) and (5),

$$F_s = S_{av} \cdot M_d/[(I/K_s) - 1] \qquad I \geq K_s \qquad (6)$$

Equation 6 can be used to predict the amount of infiltration prior to runoff and the time to the beginning of runoff, which is F_s/I. Intuitively, one sees that (6) has the correct form, for, if $M_d = 0$, the soil is saturated, and there is no infiltration prior to runoff. If $I = K_s$, then $F_s = \infty$, as it should, since all the rainfall at this low intensity will infiltrate.

A relationship similar to (6) was proposed for the intake volume to surface saturation by *Rubin and Steinhardt* [1964] for a soil with a well-defined 'bubbling pressure.' In their case, S_{av} was taken to be equal to the bubbling pres-

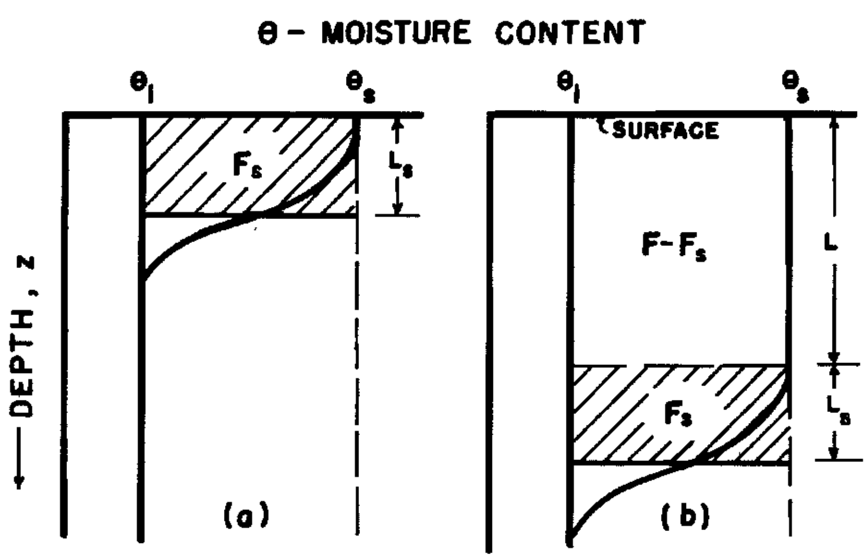

Fig. 2. Generalized soil moisture profiles during infiltration at (*a*) the moment of surface saturation and (*b*) a later time.

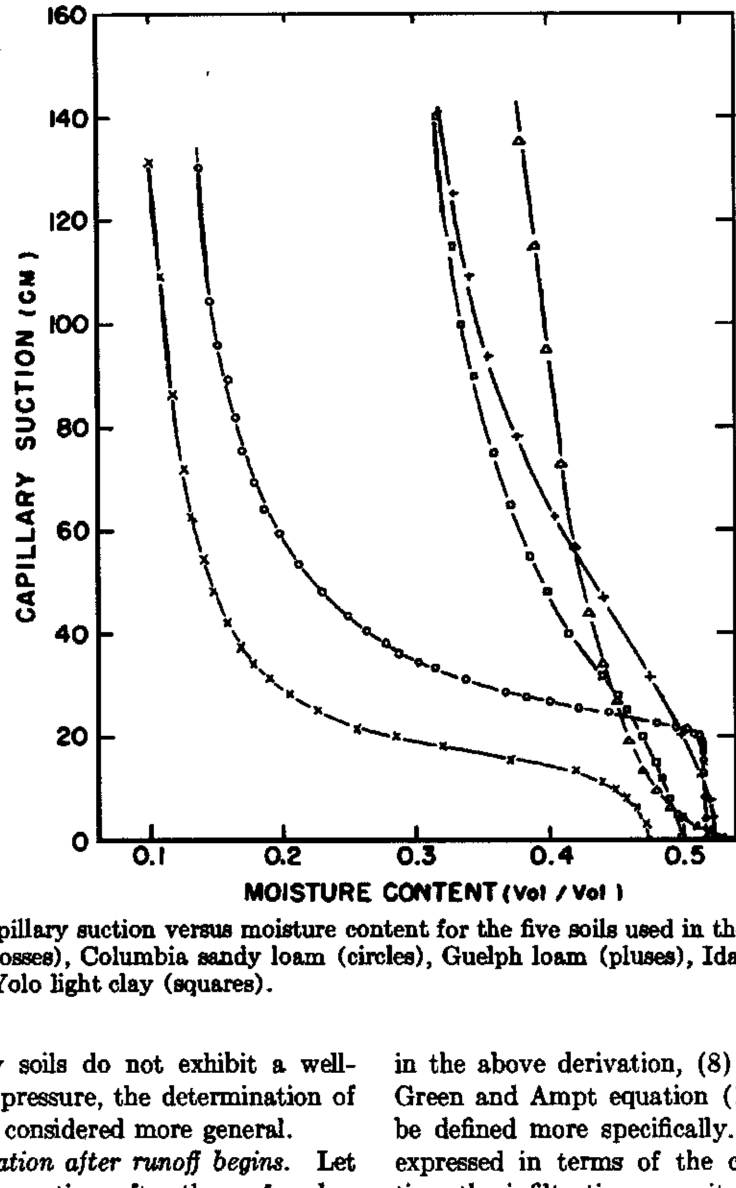

Fig. 3. Capillary suction versus moisture content for the five soils used in the study: Plainfield sand (crosses), Columbia sandy loam (circles), Guelph loam (pluses), Ida silt loam (triangles), and Yolo light clay (squares).

sure. Since many soils do not exhibit a well-defined bubbling pressure, the determination of S_{av} given later is considered more general.

Case C: infiltration after runoff begins. Let us assume that, some time after the surface has become saturated, the moisture profile can be represented by Figure 2*b*. Darcy's law can be applied again, the infiltration rate now being equal to the infiltration capacity f_p. If (4) is applied to the situation of Figure 2*b*,

$$f_p = K_s(S_{av} + L_s + L)/(L_s + L) \qquad (7)$$

If L_s is as before, $L_s = F_s/M_d$, where F is cumulative infiltration at any time and F_s is its value at the moment of surface saturation. Similarly, $L = (F - F_s)/M_d$. Hence $L_s + L = F/M_d$, and we obtain

$$f_p = K_s[1 + (S_{av} \cdot M_d/F)] \qquad (8)$$

Note that, despite the different conditions used in the above derivation, (8) is identical to the Green and Ampt equation (1), although S will be defined more specifically. We see also that, expressed in terms of the cumulative infiltration, the infiltration capacity is independent of the infiltration volume to saturation. On the other hand, relating infiltration rate to time yields a family of curves for different rainfall intensities.

One can substitute dF/dt for f_p in (8), separate the variables, and integrate to obtain an expression for cumulative infiltration as a function of time [*Mein and Larson*, 1971]. The lower limits of integration are F_s and t_s, which therefore appear in the resulting equation as constants.

Capillary suction at the wetting front. Because we are interested in the moving front, we propose that the average suction at the wetting front be determined from the S versus K relationship for the soil. At the trailing edge of the front, S is close to the saturated value (0). For the remainder of the front, S is much greater than the saturated value. The parameter S_{av} can be approximated by integrating across the front over the range of moisture content θ_i–θ_s instead of over depth. Before a rainfall event the moisture content is frequently low, and, if it is, the conductivity is very low (Figures 3 and 4). Assuming that the conductivity for θ_i is negligible (for this purpose), one can integrate as follows over the full range of moisture content:

$$S_{av} = \int_0^1 S \, dk_r \qquad (9)$$

where k_r is the relative conductivity, equal to K/K_s (Figure 4). This is simply the area under the S-k_r curve between $k_r = 0$ and $k_r = 1$. Because the suction very close to $k_r = 0$ becomes very large, for this study the range $k_r =$ 0.01–1 was used with (9).

GENERATION OF INFILTRATION DATA

In the absence of suitable field data it was decided to test the model against solutions generated by numerical solution of the Richards equation (2). An implicit iterative finite difference scheme was used, identical in most respects to that presented by *Smith* [1970] (see also *Smith and Woolhiser* [1971]). Full details of the methods used in this study are given elsewhere [*Mein and Larson*, 1971].

A variety of soils for which the suction-conductivity–moisture content curves were available were selected from published data.

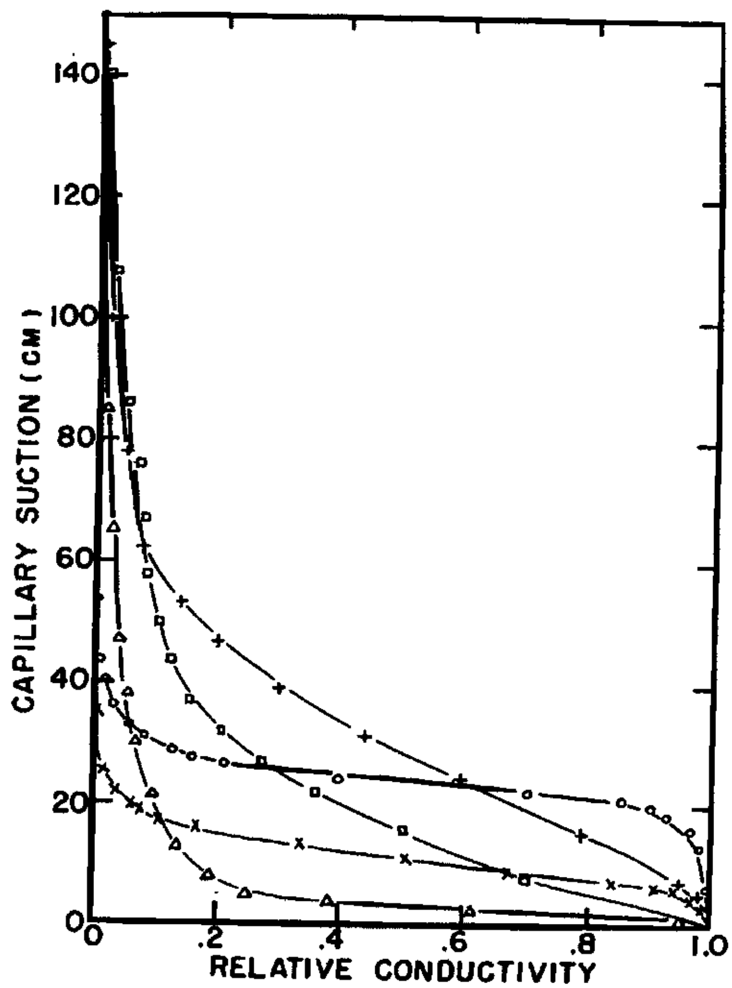

Fig. 4. Capillary suction versus relative conductivity for the five soils used in the study, symbolized as in Figure 3.

TABLE 1. Soils Selected for the Infiltration Study

Soil	K_s, cm/sec	Porosity
Plainfield sand (disturbed sample) [*Black et al.*, 1969]	3.44×10^{-3}	0.477
Columbia sandy loam (disturbed sample) [*Laliberte et al.*, 1966]	1.39×10^{-3}	0.518
Guelph loam (air-dried, sieved) [*Elrick and Bowman*, 1964]	3.67×10^{-4}	0.523
Ida silt loam (undisturbed sample) [*Green*, 1962]	2.92×10^{-5}	0.530
Yolo light clay (disturbed sample) [*Moore*, 1939]	1.23×10^{-5}	0.499

The soils chosen ranged in texture from a sand to a light clay (Table 1). The published data for the first three soils in Table 1 were desorption data, but, since infiltration is an absorption process, it is important to use the absorption or wetting curve of the soil characteristic curves. For coarse soils a rough approximation for most of the wetting curve can be obtained by dividing the drying curve suction scale by 1.6 (D. A. Farrell, private communication, 1970). This was done for the sand, sandy loam, and loam.

The suction versus moisture content and suction versus relative conductivity curves used in the Richards equation to generate infiltration data are shown in Figures 3 and 4. A wide variation in soil characteristics is represented by these curves.

Several combinations of rainfall intensity and initial moisture content were chosen for each soil type. Rainfall intensities of 4 and 8 times the saturated conductivity were used for each soil. The highest initial moisture content was chosen such that the relative conductivity was small (corresponding perhaps to field capacity), the remaining values being spread over the remaining range of the soil data. Columbia sandy loam and Ida silt loam were selected for more extensive testing. For these soils, rainfall intensities of 2 and 6 times the saturated conductivity were added, together with extra levels of moisture content. The various combinations used in the study are given in Table 2.

Examples of the data generated by numerical solution of (2) are shown in Figures 5 and 6. The results of all 40 runs were presented earlier [*Mein and Larson*, 1971]. The two sets of curves given here (and others) show clearly the influence of rainfall intensity and initial moisture content. Errors due to the finite difference formulation of (2) were determined by continuity and, by modifying the finite difference grid size in preliminary runs, were reduced to <1%. To obtain this degree of accuracy, depth increments were varied from 0.2 cm near the surface to 5.0 cm.

EVALUATION OF MODEL

Case B: infiltration prior to runoff. To predict the infiltration volume prior to surface saturation F_s, (6) is used. However, S_{av} must first be evaluated from (9) by using the S-k_r curves for each soil (Figure 4) from $k_r = 0.01$ to $k_r = 1.0$. The values obtained for each soil, in the same order as in Table 2, are 11.7, 23.8, 31.4, 7.4, and 22.4 cm.

The appropriate value of S_{av} and the other variables being substituted into (6), the volumes of infiltration to surface saturation were predicted for all runs and compared to the values computed by using (2). The comparison is made in Figure 7. Although the errors for small values of F_s in Figure 7 appear significant (i.e., relative to the total amounts), the errors themselves are generally <0.2 cm.

TABLE 2. Values of Relative Rainfall Intensity I/K_s and Initial Moisture Content M_c

Soil	I/K_s	M_c*	No. of Events
Plainfield sand	4, 8	0.13, 0.23	4
Columbia sandy loam	2, 4, 6, 8	0.125, 0.15, 0.20, 0.25, 0.318	14
Guelph loam	4, 8	0.30, 0.35	4
Ida silt loam	2, 4, 6, 8	0.25, 0.30, 0.35, 0.40, 0.43	14
Yolo light clay	4, 8	0.25, 0.35	4

* Volume basis.

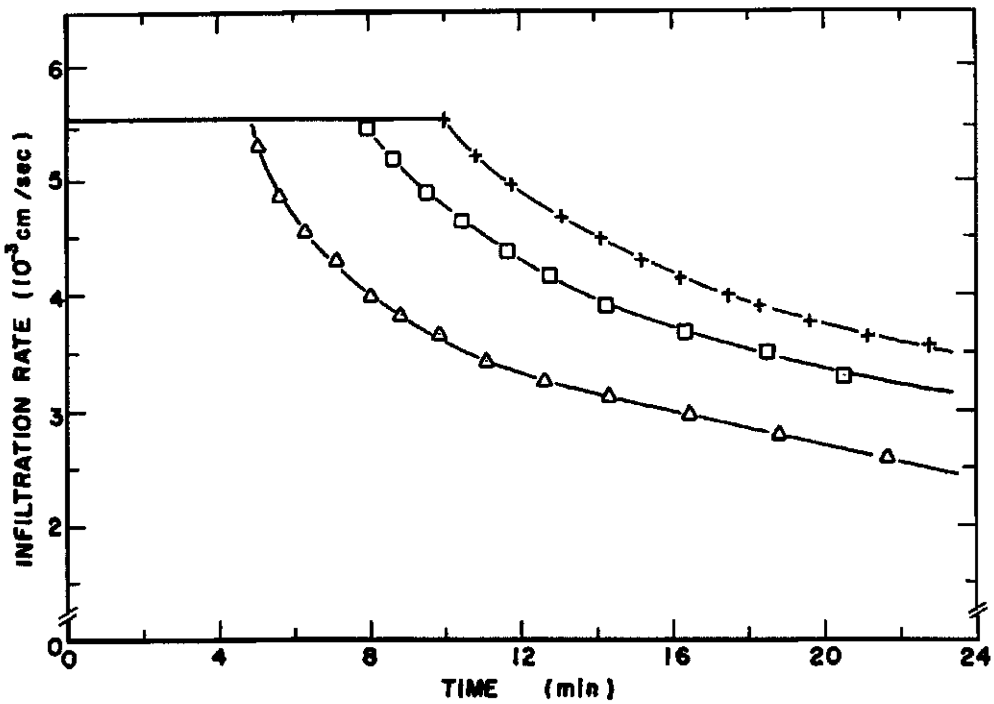

Fig. 5. Plot of infiltration rate versus time for Columbia sandy loam showing the effect of initial moisture content, computed by (2). For all tests, $I = 4K_s$; $I = 5.56 \times 10^{-3}$ cm/sec; $K_s = 1.39 \times 10^{-3}$ cm/sec. Values of M_c are 0.125 (pluses), 0.200 (squares), and 0.318 (triangles).

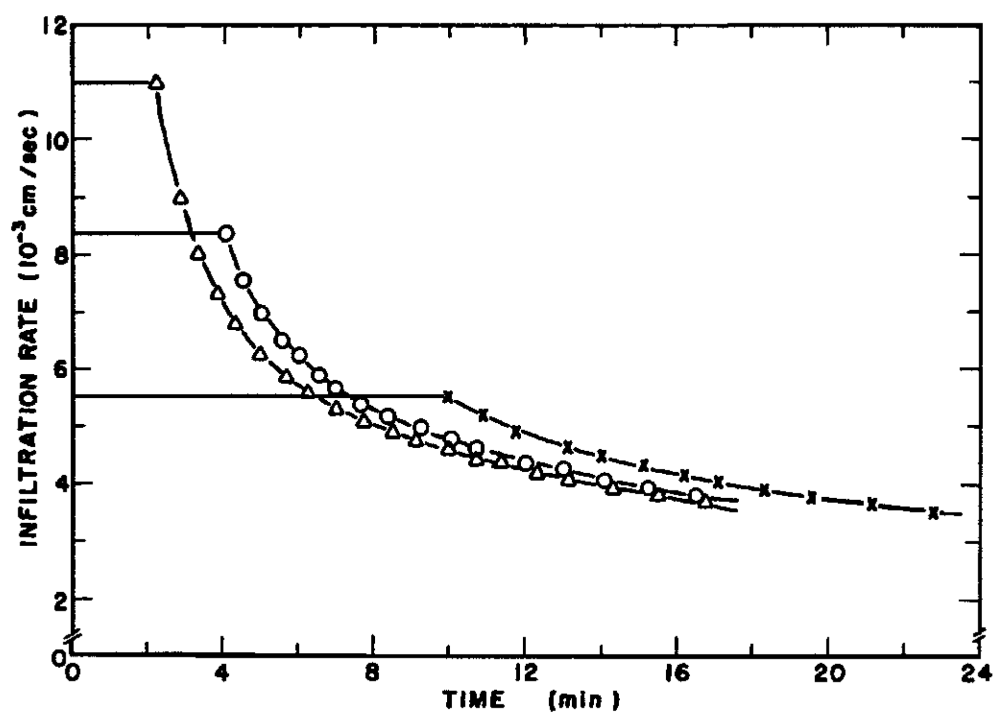

Fig. 6. Plot of infiltration rate versus time for Columbia sandy loam showing the effect of rainfall intensity, computed by (2). For all cases $M_c = 0.125$; $K_s = 1.39 \times 10^{-3}$ cm/sec. Values of I are $4K_s$ (crosses), $6K_s$ (circles), and $8K_s$ (triangles).

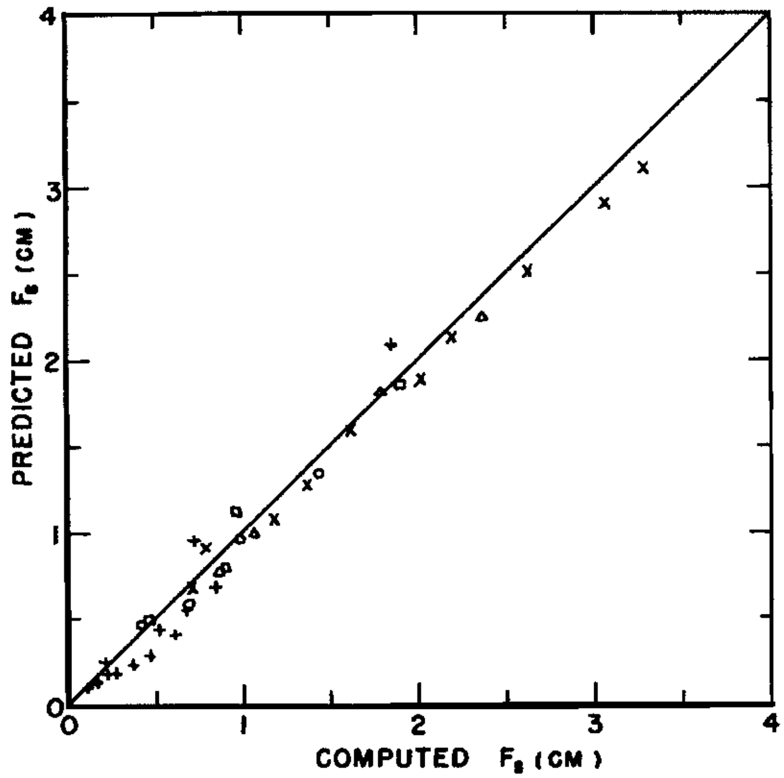

Fig. 7. Comparison of the predicted volume of infiltration at surface saturation (6) to the volume of surface saturation computed by solution of the Richards equation. Circles represent Plainfield sand; crosses, Columbia sandy loam; triangles, Guelph loam; pluses, Ida silt loam; squares, Yolo light clay.

A further test of (6) is possible by using the experimental data of *Rubin and Steinhardt* [1964] for rainfall infiltration into columns of air dry Rehovot sand. Because the actual moment of surface saturation (assumed in this study to be the moment when the capillary suction at the surface just becomes 0) is difficult to determine, Rubin and Steinhardt noted three stages: visible retardation of rainfall just apparent (stage A), one third of surface just covered by water (stage P), and surface just completely covered (stage C). They reasoned that surface saturation should occur somewhere between stages A and P, and their experimental points for these stages are shown in Figure 8.

From their published data for Rehovot sand the following properties were noted: the porosity is 0.387 (volume/volume), the saturated conductivity is 47.9 cm/hr, the initial moisture content (air dry) was taken to be 0.025 (volume/volume), and the average capillary suction was computed to be 16.1 cm. The values of F_s predicted by (6) for the appropriate rainfall intensities are plotted along with the values observed by Rubin and Steinhardt (Figure 8). The results provide further evidence of the validity of (6).

Case C: prediction of infiltration capacity after runoff begins. This part of the model could be tested in two ways: either by using the moment of saturation computed from the Richards equation as the starting point (an independent test) or by using the starting point predicted by the first stage of the model (6). Because (6) is a more severe test than the Richards equation and because it is a necessary part of the infiltration model, we decided to use the second approach.

For comparison, infiltration rates and cumulative infiltration amounts predicted by (8) for each of the 40 runs were tabulated along with those computed by solving the Richards equation. Since the data are too voluminous to present here, two or three well-spaced points on each infiltration curve were selected arbitrarily and plotted. The agreement between predicted and calculated infiltration amounts (Figure 9) is quite good and in fact better than the agreement between predicted and calculated amounts to surface saturation (Figure 7). The infiltration rates (Figure 10) do not agree as well as the infiltration amounts, as one would expect, but can be considered satisfactory in most cases. These errors do not appear to be serious, since the cumulative infiltration volumes are in good agreement.

On a percentage basis the errors in computing F were more apparent and varied considerably between soil types. For the first three soils (Table 2) the percentage errors were small, usually under 3% and never exceeding 5%. For the Yolo soil the errors were somewhat higher but usually under 10%. The errors for Ida silt loam were well over 10% for a number of cases. An inspection of Figure 9, however, shows that the total amounts of infiltration for both the Ida silt loam and the Yolo clay loam were small, relatively high percentage errors thus being produced, and that the absolute magnitudes of the errors were generally less than 0.2 cm for all the soils. Thus from the hydrologist's point of view the predicted values appear to be satisfactory in all cases.

CONCLUSION

A two-stage model is proposed for representing infiltration as a function of measurable soil properties, initial moisture content, and rainfall intensity. The first stage is a period of no rainfall excess, whose duration is determined by using (6). In the second stage, infiltration occurs at infiltration capacity, given by (8), and is accompanied by a rainfall excess.

Tests of the model with a wide variety of soil types show that it has good to excellent predictive ability for the conditions assumed in the study. These are constant rainfall intensity, homogeneous soil, and uniform initial moisture content in the zone of infiltration. An additional requirement is that the relationships of soil moisture tension and hydraulic conductivity to moisture content are known or determinable.

It is evident that the above conditions are probably never fully satisfied. The effects of departures from these idealized conditions were not within the scope of the study reported here.

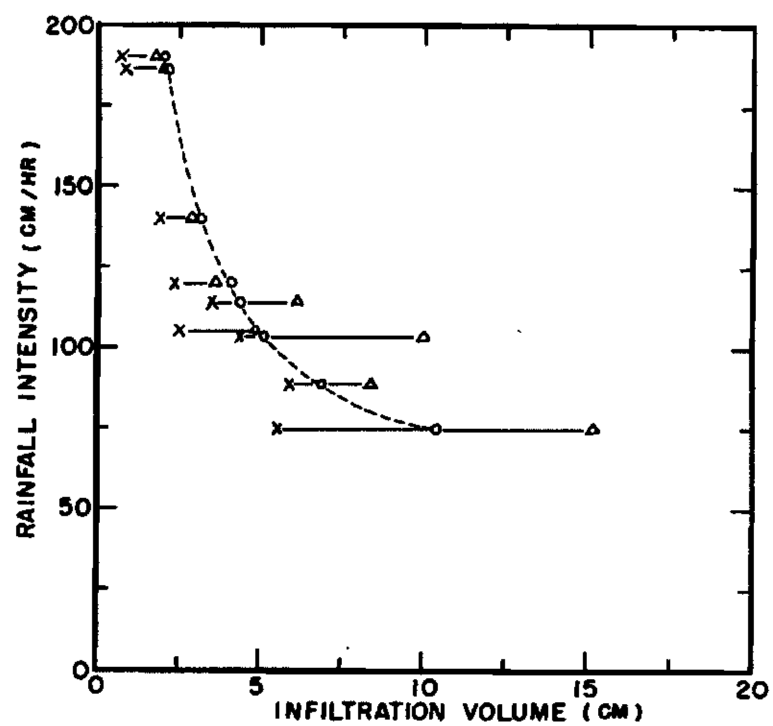

Fig. 8. Comparison of predicted (6) and observed values of infiltration volume at surface saturation F_s for columns of Rehovot sand (observed data by *Rubin and Steinhardt* [1964]). Circles indicate values predicted from (6); crosses, values obtained during experimental stage A; and triangles, values obtained during experimental stage P.

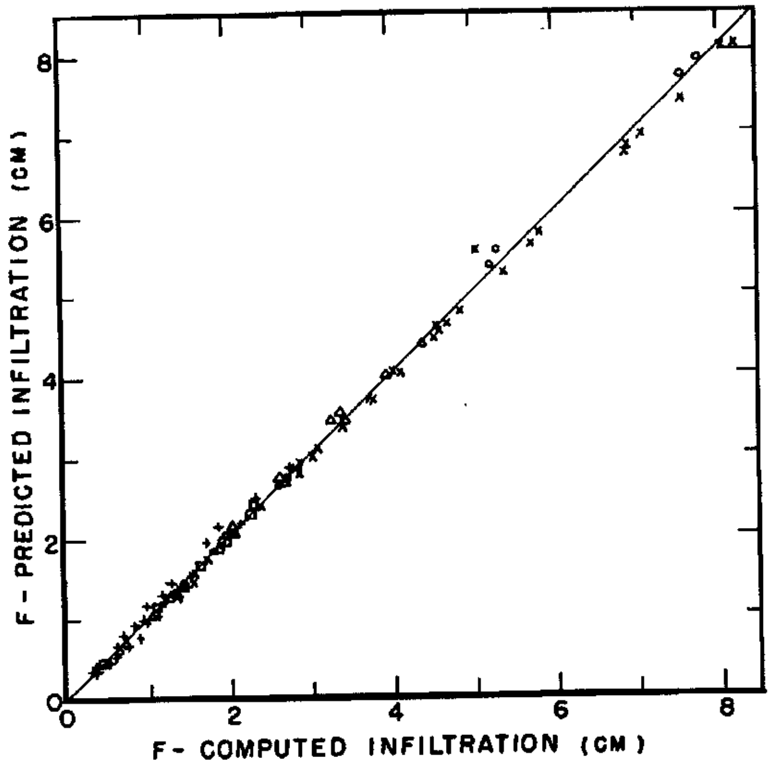

Fig. 9. Comparison of predicted (8) and computed (Richards equation) values of cumulative infiltration F for all soils and all tests. The five soils used are symbolized as in Figure 7.

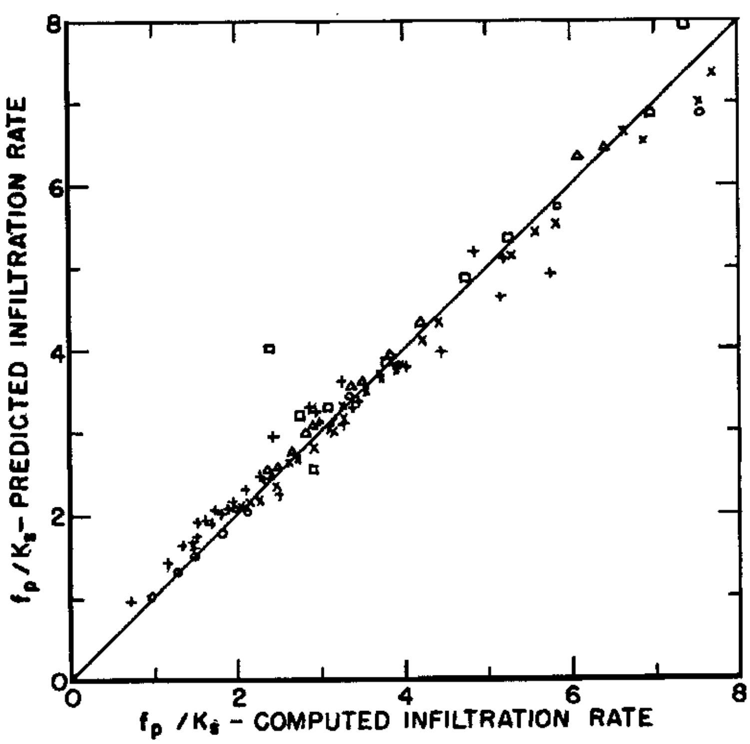

Fig. 10. Comparison of predicted (8) and computed (Richards equation) values of the relative infiltration rate f_p/K_s for all soils and all tests. The five soils used are symbolized as in Figure 7.

Thus, until further studies are made, they represent limitations or uncertainties that should be kept in mind.

The model has several noteworthy features. First, it represents the actual infiltration process and therefore predicts infiltration as a function of measurable soil characteristics, currently for rather limited conditions but potentially for a wider range. Empirical infiltration equations and models, on the other hand, require the use of fitted parameters. Second, the model is applicable to events that produce a delayed rainfall excess. Most empirical models are not. Finally, applying the model to individual rainfall events involves very simple calculations comparable to those with common infiltration equations.

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REFERENCES

Black, T. A., W. R. Gardner, and G. W. Thurtell, The prediction of evaporation, drainage, and soil water storage for a bare soil, *Soil Sci. Soc. Amer. Proc., 33*(5), 655–660, 1969.

Bouwer, H., Rapid field measurement of air-entry value and hydraulic conductivity of soil as significant parameters in flow system analysis, *Water Resour. Res., 2*(4), 729–738, 1966.

Bouwer, H., Infiltration of water into non-uniform soil, *J. Irrig. Drain. Div. Amer. Soc. Civil Eng., 95*(IR4), 451–462, 1969.

Childs, E. C., *An Introduction to the Physical Basis of Soil and Water Phenomena,* pp. 210–212, 277–279, John Wiley, New York, 1969.

Childs, E. C., and M. Bybordi, The vertical movement of water in a stratified porous material, 1, Infiltration, *Water Resour. Res., 5*(2), 446–459, 1969.

Chow, V. T., Hydrology and its development, in *Handbook of Hydrology,* edited by V. T. Chow, sect. 1, p. 4, McGraw-Hill, New York, 1964.

Crawford, N. H., and R. K. Linsley, The synthesis of continuous streamflow hydrographs on a digital computer, *Tech. Rep. 12,* pp. 28–52, Dep. of Civil Eng., Stanford Univ., Stanford, Calif., 1962.

Elrick, D. E., and D. H. Bowman, Note on an improved apparatus for soil moisture flow measurements, *Soil Sci. Soc. Amer. Proc., 28*(3), 450–453, 1964.

Green, R. E., Infiltration of water into soils as influenced by antecedent moisture, Ph.D. thesis, Iowa State Univ., Ames, 1962.

Green, W. H., and G. A. Ampt, Studies on soil physics, 1, The flow of air and water through soils, *J. Agr. Sci., 4*(1), 1–24, 1911.

Hillel, D., and W. R. Gardner, Transient infiltration into crust-topped profiles, *Soil Sci., 109*(2), 69–76, 1970.

Holtan, H. N., A concept for infiltration estimates in watershed engineering, *Pap. 41-51,* pp. 10–12, Agr. Res. Serv., U.S. Dep. of Agr., Beltsville, Md., 1961.

Huggins, L. F., and E. J. Monke, The mathematical simulation of the hydrology of small watersheds, *Tech. Rep. 1,* 130 pp., Water Resour. Res. Center, Purdue Univ., Lafayette, Ind., 1966.

Laliberte, G. E., A. T. Corey, and R. H. Brooks, Properties of unsaturated porous media, *Hydrol. Pap. 17,* 40 pp., Colo. State Univ., Fort Collins, 1966.

Mein, R. G., and C. L. Larson, Modeling of the infiltration component of the rainfall-runoff process, *Bull. 43,* pp. 21–29, Water Resour. Res. Center, Univ. of Minn., Minneapolis, 1971.

Moore, R. E., Water conduction from shallow water tables, *Hilgardia, 12*(6), 383–426, 1939.

Nielsen, D. R., D. Kirkham, and W. R. Van Wijk, Diffusion equation calculations of field soil water infiltration profiles, *Soil Sci. Soc. Amer. Proc., 25*(3), 165–168, 1961.

Philip, J. R., The theory of infiltration, 4, Sorptivity and algebraic infiltration equations, *Soil Sci., 84*(3), 257–264, 1957.

Rubin, J., and R. Steinhardt, Soil water relations during rain infiltration, 3, Water uptake at incipient ponding, *Soil Sci. Soc. Amer. Proc., 28*(5), 614–619, 1964.

Smith, R. E., Mathematical simulation of infiltrating watersheds, Ph.D. thesis, Colorado State Univ., Fort Collins, 1970.

Smith, R. E., and D. E. Woolhiser, Overland flow on an infiltrating surface, *Water Resour. Res., 7*(4), 899–913, 1971.

Wang, F. C., and V. Lakshminarayana, Mathematical simulation of water movement through unsaturated non-homogeneous soils, *Soil Sci. Soc. Amer. Proc., 32*(3), 329–334, 1968.

Whisler, F. D., and H. Bouwer, Comparison of methods for calculating vertical drainage and infiltration in soils, *J. Hydrol., 10*(1), 1–19, 1970.

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